WIMS'11 Tutorial

Sogndal, May 2011

Description Logics Reasoning for the Semantic Web

Anni-Yasmin Turhan

TU Dresden, Germany

TU
Dresden

Semantic Web

Idea:

- resources such as data, services etc. are labelled with annotations
- annotations describe properties of the resource
- annotations: machine processable

Ontologies for annotations:

- agreed vocabulary
- descriptions of categories relevant to the application

W3C:
Web ontology language (OWL) recommended as the ontology language for the semantic web

Reasoning for the Semantic Web

- Intelligent access to resources
  :use information that follows from the descriptions
- Extract implicitly captured information by reasoning
  – automatically!
- Requires formal semantics of ontology formalism

Description Logics (DLs)

- are a family of different logics with varying expressivity
- knowledge representation formalism with formal semantics
- formalism for declarative description of facts
- come with powerful reasoning services:
  make implicit knowledge explicit
- are applied in many practical areas
- are the basis for the web ontology language OWL!

Description Logics research

- investigates reasoning algorithms of DLs
- emphasis on sound, complete and terminating reasoning algorithms
  ➵ guarantees for the reasoning service
- since late nineties:
  - efficient DL reasoning systems available for DLs
    for which reasoning is not tractable
  - investigation of very expressive (but computationally costly) DLs
- since last decade:
  - investigation of DLs that have low expressivity,
    but allow for very efficient reasoning

The OWL standard

OWL 1:

- W3C recommendation of 2004
- OWL DL and OWL Lite: DL-based ontology languages

OWL 2:

- W3C recommendation of 2009
- consists of
  - an expressive language
  - 2 profiles that correspond to light-weight DLs

Tutorial overview

1. DLs: Basic notions
   1.1. concept descriptions,
   1.2. TBox,
   1.3. ABox

   and their reasoning services
2. Reasoning in $\mathcal{ALC}$
3. OWL 2: beyond $\mathcal{ALC}$
4. OWL 2 EL
   4.1 the $\mathcal{EL}$-family
   4.2. Reasoning method for $\mathcal{EL}$
5. OWL 2 QL
   5.1. DL-Lite family
   5.2. Conjunctive queries
6. Conclusions

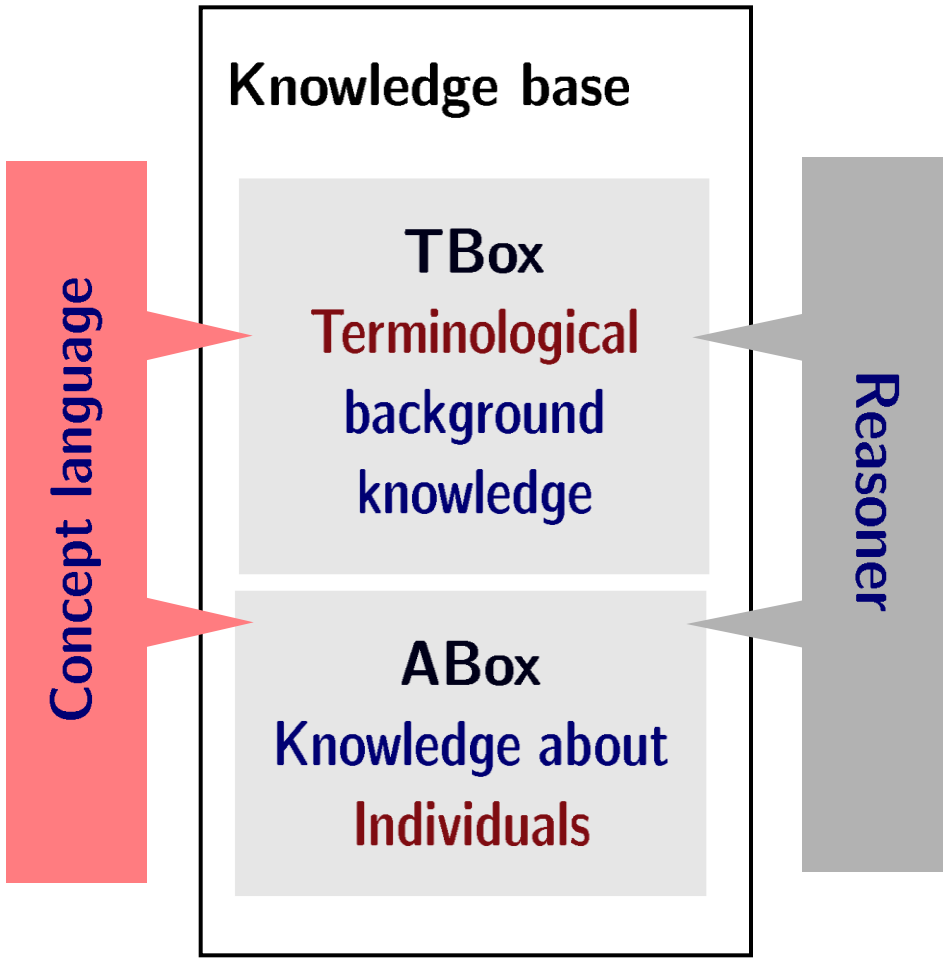
DL knowledge bases

DL knowledge bases have two parts:

1. Representation of the intensional knowledge
   - Representation of categories and relations from the domain of discourse
   - Terminological knowledge stored in the TBox

2. Representation of extensional knowledge
   - Representation of individuals and related tuples from the domain
   - Assertional knowledge stored in the ABox

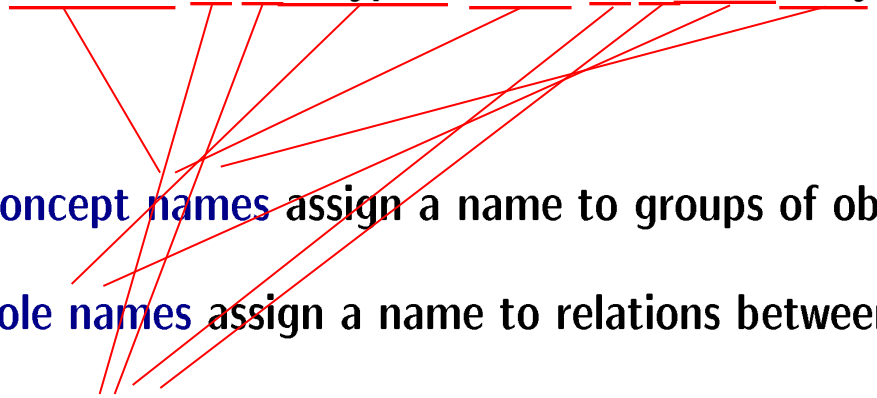
Overview DL systems



Defining Concepts with DLs

The core part of any DL is the concept language

$$\text{Mammal} \sqcap \exists \text{bodypart}.\text{Trunk} \sqcap \forall \text{color}.\text{Grey}$$



- concept names assign a name to groups of objects
- role names assign a name to relations between objects
- constructors allow to related concept names and role names

Complex concepts can be used in concept definitions:

$$\text{Elephant} \equiv \text{Mammal} \sqcap \exists \text{bodypart}.\text{Trunk} \sqcap \forall \text{color}.\text{Grey}$$

A set of such concept definitions is an ontology

The Description Logic $\mathcal{EL}$: Syntax

Atomic types: concept names $A, B, \ldots$ (unary predicates)
role names $R, S, \ldots$ (binary predicates)

Constructors: $C \sqcap D$ (conjunction)

$\exists R.C$ (existential restriction)

Special concept: $\top$ (top concept)

For example: Person $\sqcap$ $\exists$ has-child.Person

The Description Logic $\mathcal{ALC}$: Syntax

Atomic types: concept names $A, B, \ldots$ (unary predicates)

role names $R, S, \ldots$ (binary predicates)

Constructors:

$\neg C$	(negation)
$C \sqcap D$	(conjunction)
$C \sqcup D$	(disjunction)
$\exists R.C$	(existential restriction)
$\forall R.C$	(value restriction)

Special concepts:

$\top$	(top concept)
$\bot$	(bottom concept)

For example: $\neg(\ A\ \sqcup\ \exists R.(\forall S.B \sqcap \neg A))$

$\mathsf{Mammal} \sqcap \exists \mathsf{bodypart}.\mathsf{Trunk} \sqcap \forall \mathsf{color}.\mathsf{Grey}$

Example: $\mathcal{ALC}$ Concept Descriptions

Signature: $N_C = \{$ Person, Male, Happy $\}$,

$N_R = \{$has-child, has-sibling, likes, knows $\}$

Parent $\equiv$

Person $\sqcap$ $\exists$ has-child.Person

Grandparent $\equiv$

Person $\sqcap$ $\exists$ has-child.($\exists$ has-child. Person)

Uncle-of-happy-children $\equiv$

Person $\sqcap$ Male $\sqcap$ $\exists$ has-sibling.($\exists$ has-child.Person)
$\sqcap$ $\forall$ has-sibling.($\forall$ has-child.Happy)

Interpretations

Semantics based on interpretations $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where

- $\Delta^{\mathcal{I}}$ is a non-empty set (the domain)
- $\cdot^{\mathcal{I}}$ is the interpretation function mapping
  each concept name A to a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$ and
  each role name R to a binary relation $R^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$.

Semantics of Primitive Concepts

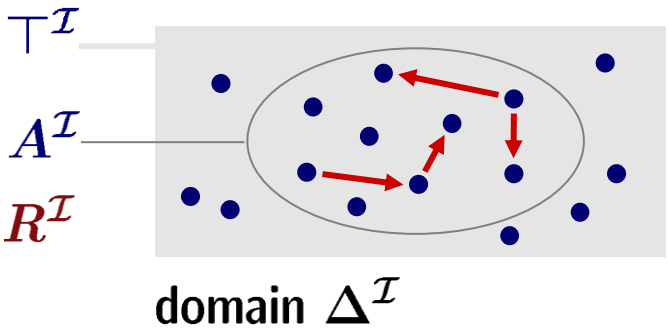
Semantics based on interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Concepts: Subsets of domain $\Delta^{\mathcal{I}}$

Roles: binary relations on domain $\Delta^{\mathcal{I}}$

Primitive concepts

$$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$$
$$\bot^{\mathcal{I}} = \emptyset$$
$$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$$



Complex $\mathcal{ALC}$-Concepts: Semantics

Semantics of complex concepts:

$$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$$

$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$$

$$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$

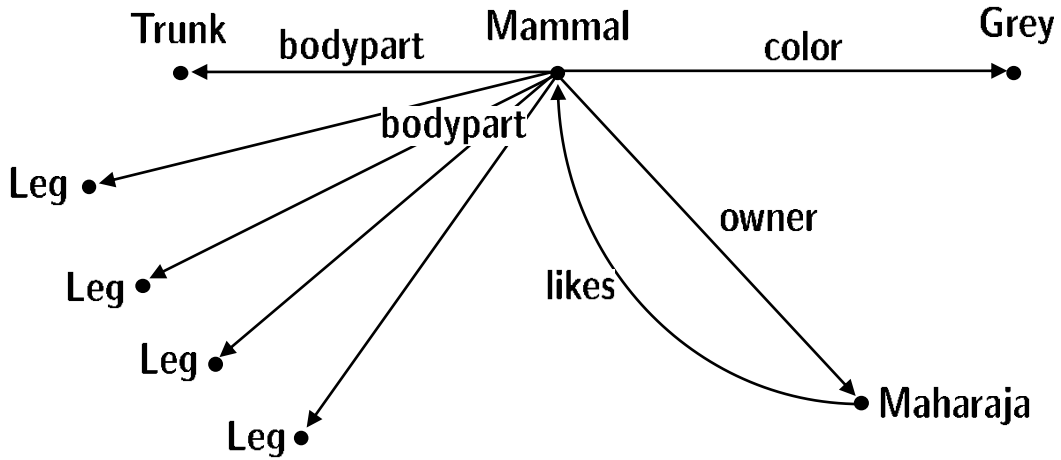
$$(\exists R.C)^{\mathcal{I}} = \{d \mid \text{there is an } e \in \Delta^{\mathcal{I}} \text{ with } (d,e) \in R^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\}$$

$$(\forall R.C)^{\mathcal{I}} = \{d \mid \text{for all } e \in \Delta^{\mathcal{I}}, (d,e) \in R^{\mathcal{I}} \text{ implies } e \in C^{\mathcal{I}}\}$$

Example: semantics of complex concepts

$$\mathsf{Mammal} \sqcap \exists \mathsf{bodypart}.\mathsf{Trunk} \sqcap \forall \mathsf{color}.\mathsf{Grey}$$

$$\mathsf{Mammal} \sqcap \forall \mathsf{bodypart}.(\mathsf{Trunk} \sqcup \mathsf{Leg})$$



Reasoning tasks for concepts

1. Concept satisfiability — does there exist a model of C?

   model of C: interpretation $\mathcal{I}$ with $C^{\mathcal{I}} \neq \emptyset$

   If unsatisfiable, the concept contains a contradiction.

2. Concept subsumption — does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold for all $\mathcal{I}$?

   written $C \sqsubseteq D$

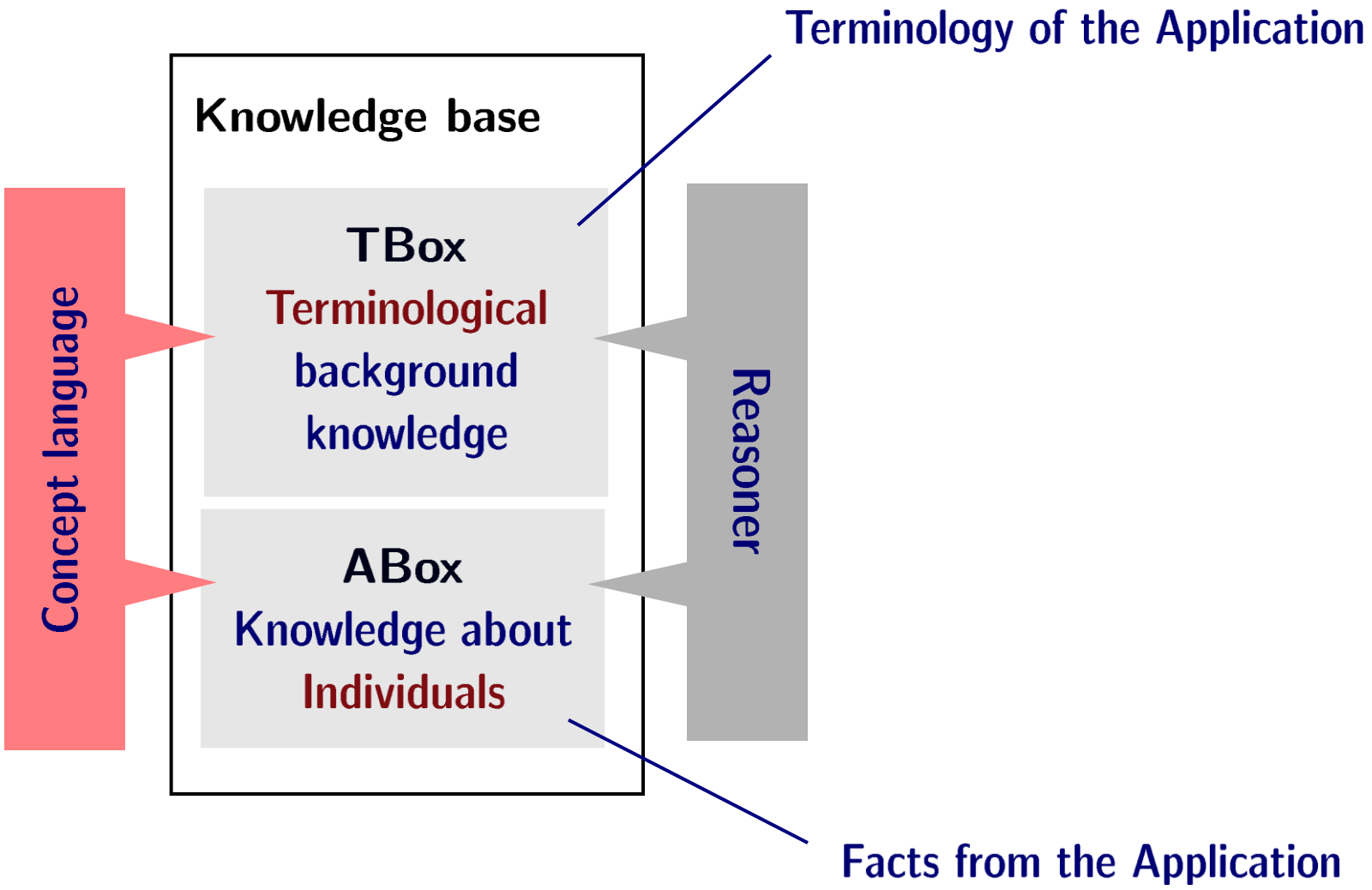
   If $C \sqsubseteq D$, then D is more general than C

Examples

- $\forall \text{owner}.\text{Rich} \sqcap \forall \text{owner}.\text{Famous} \sqsubseteq \forall \text{owner}.(\text{Rich} \sqcap \text{Famous})$
- Set $\top := A \sqcup \neg A$. Then $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$ for all $\mathcal{I}$.
  Thus $C \sqsubseteq \top$ for all C.
- Set $\bot := A \sqcap \neg A$. Then $\bot^{\mathcal{I}} = \emptyset$ for all $\mathcal{I}$.
  Thus $\bot \sqsubseteq C$ for all C.
- $C \sqsubseteq D$ if and only if $C \sqcap \neg D$ is not satisfiable
- C is satisfiable if not $C \sqsubseteq \bot$.

➳ Subsumption can be reduced to (un)satisfiability and vice versa.

DL systems are more than a concept language



TBox: Motivation

TBoxes capture:

- Intensional knowledge
- Categories from the application domain in terms of concepts
- Relations from the application domain in terms of roles
- Relations between concepts
  e.g. assign names to complex concepts or
  state super- / sub-concept relationships

TBox: Syntax

TBox $\mathcal{T}$: Finite set of concept axioms.

There are different kinds of concept axioms:

- Primitive concept definition
  $A \sqsubseteq D \qquad A \in N_C$
  ➳ necessary conditions
- Concept definition
  $A \equiv D \qquad A \in N_C$
  ➳ necessary & sufficient conditions
- Concept disjointness axiom
  $C \sqcap D \sqsubseteq \bot$
- General concept equivalence
  $C \equiv D$
- General concept (inclusion) axiom (GCI)
  $C \sqsubseteq D$

GCIs can express other concept axioms

- Primitive concept definition: $A \sqsubseteq D$ $\quad A \in N_C$

  Pick a concept name for left-hand side.

- Concept definition: $A \equiv D$ $\quad A \in N_C$

  $A \sqsubseteq D, \quad D \sqsubseteq A$

- Concept disjointness axiom : $C \sqcap D \sqsubseteq \bot$

  Is a GCI.

- General concept equivalence: $C \equiv D$

  $C \sqsubseteq D, \quad D \sqsubseteq C$

Kinds of TBoxes

1. TBox $\mathcal{T}$ is a general TBox, if
   - it is a finite set of concept axioms
   - cyclic definitions and GCIs are allowed

$$\{\mathsf{WildAnimal} \equiv \mathsf{Animal} \sqcap \neg\exists \mathsf{owner}.\top,$$
$$\mathsf{Mammal} \sqcap \exists \mathsf{bodypart}.\mathsf{Hunch} \equiv \mathsf{Camel} \sqcup \mathsf{Dromedary}\}$$

2. TBox $\mathcal{T}$ is an unfoldable TBox, if it has
   - only (primitive) concept definitions
   - concept names at most once on the left-hand side of definitions
   - no cyclic definitions, no GCIs

$$\{\mathsf{Elephant} \equiv \mathsf{Mammal} \sqcap \exists \mathsf{bodypart}.\mathsf{Trunk}$$
$$\mathsf{Mammal} \equiv \mathsf{Elephant} \sqcup \mathsf{Lion} \sqcup \mathsf{Zebra}\}$$

➽ Unfoldable TBoxes can be conceived as macro definitions.

TBox: Semantics

Semantics of concept axioms

$A \equiv C$ holds in an interpretation $\mathcal{I}$ iff $A^{\mathcal{I}} = C^{\mathcal{I}}$

$C \sqsubseteq D$ holds in an interpretation $\mathcal{I}$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$

Semantics of the TBox

An interpretation $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ if

$C^{\mathcal{I}} = D^{\mathcal{I}}$ for all $C \equiv D \in \mathcal{T}$ and
$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all $C \sqsubseteq D \in \mathcal{T}$.

Terminological Reasoning Services

Reasoning tasks for TBoxes:

1. **Concept satisfiability w.r.t. TBoxes**

   Given C and $\mathcal{T}$, does there exist a common model of C and $\mathcal{T}$?

   "Does C state a contradiction w.r.t. $\mathcal{T}$?"

2. **Concept subsumption w.r.t. TBoxes** $(C \sqsubseteq_{\mathcal{T}} D)$

   Given C,D and $\mathcal{T}$, does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold in all models of $\mathcal{T}$?

   "Is C more specific than D w.r.t. the axioms in $\mathcal{T}$?"

3. **Concept equivalence w.r.t. TBoxes** $(C \equiv_{\mathcal{T}} D)$

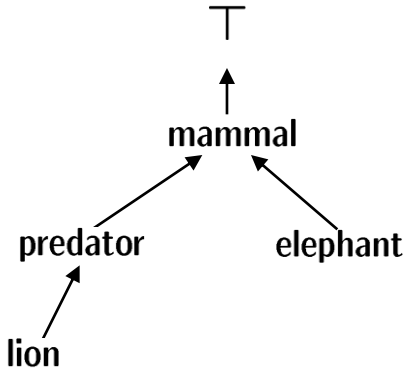
   $C \equiv_{\mathcal{T}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$ and $D \sqsubseteq_{\mathcal{T}} C$

   "Are C and D equivalent w.r.t. $\mathcal{T}$?"

4. **Classification of the TBoxes**

   Computation of all subsumption relationships between all concept names in $\mathcal{T}$.

   $\Longrightarrow$ Subsumption can be used to compute a concept hierarchy:



Example for TBox reasoning

TBox

$$\text{Elephant} \equiv \text{Mammal} \sqcap \exists \text{has-bodypart}.\text{Trunk} \sqcap \forall \text{has-color}.\text{Grey}$$

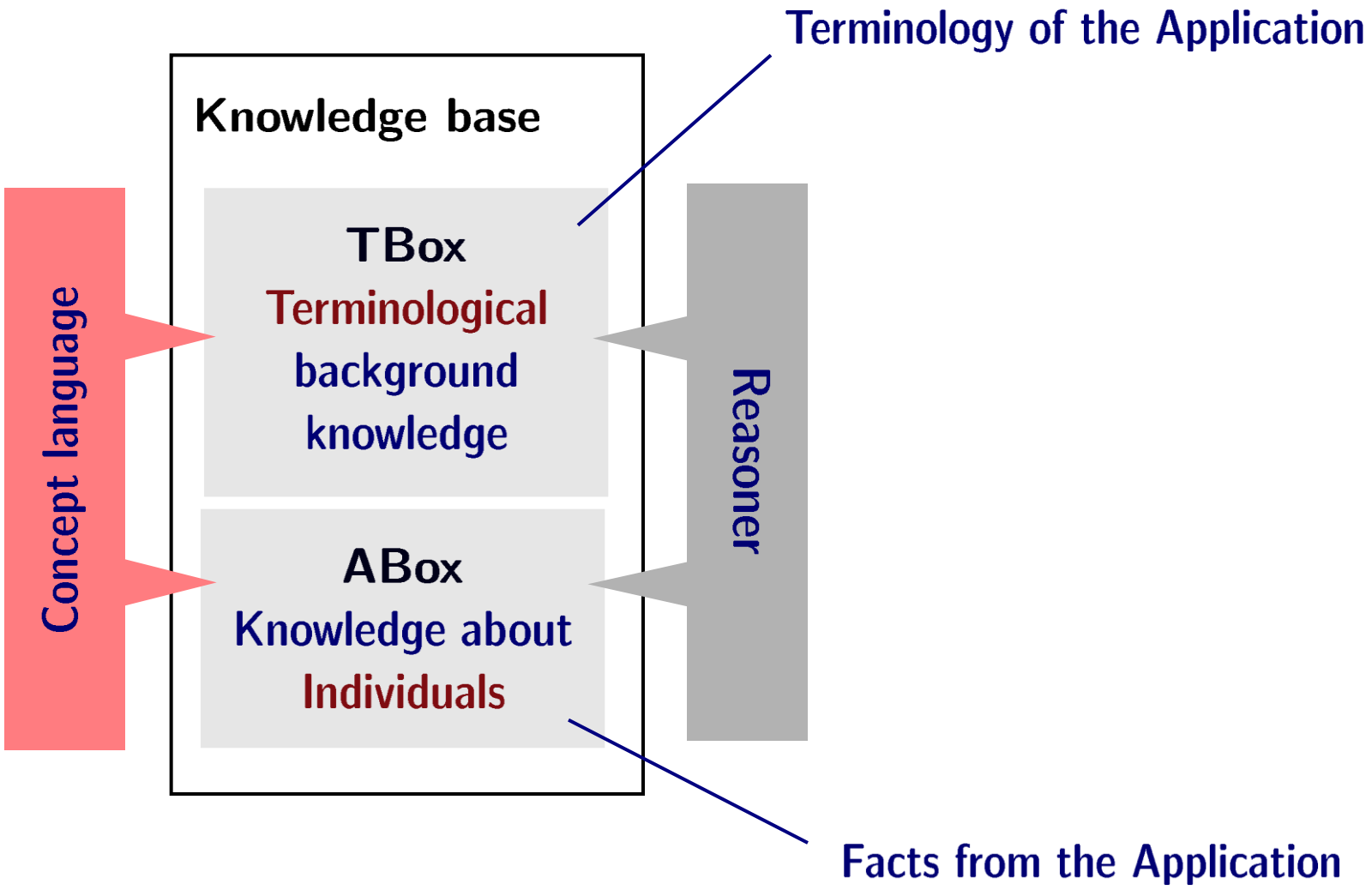
$$\text{AlbinoElephant} \equiv \text{Elephant} \sqcap \forall \text{has-color}.\text{White} \sqcap \exists \text{color}.\text{White}$$

~~$\text{Grey} \sqcap \text{White} \sqsubseteq \bot$~~

$$\text{White} \sqsubseteq \text{Grey}$$

1. TBox is consistent.
2. TBox is inconsistent.
3. AlbinoElephant $\sqsubseteq$ Elephant w.r.t. TBox.

DL systems are more than a concept language



The assertional part

ABoxes capture:

- extensional knowledge
- facts from the application domain
- knowledge about individuals
- knowledge about the relations between individuals

ABoxes: syntax

An ABox $\mathcal{A}$ is a finite set of assertions.

ABox assertions in DL systems are:

- Concept assertions: $C(a)$
- Role assertions: $(a, b) : R$
- (In)equality assertions: $a \equiv b \quad (a \not\equiv b)$

Typically, ABoxes use concept names and role names from the TBox.

ABoxes: semantics

Idea: Interpretation $\mathcal{I}$ maps each individual name a to an element of $\Delta^{\mathcal{I}}$.

Semantics of assertions:

- Concept Assertions: $\mathcal{I}$ satisfies $C(a) \iff a^{\mathcal{I}} \in C^{\mathcal{I}}$
  "$\mathcal{I}$ satisfies $C(a)$ iff the extension of a is in the extension of C."

- Role Assertions: $\mathcal{I}$ satisfies $(a, b) : R \iff (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$
  "$\mathcal{I}$ satisfies $(a, b) : R$ iff the extension of a and b is in the extension of R."

- (In)equality assertions: $\mathcal{I}$ satisfies $a \not\equiv b \iff a^{\mathcal{I}} \not\equiv b^{\mathcal{I}}$
  "$\mathcal{I}$ satisfies $a \not\equiv b$ iff the extension of a is (not) in the extension of b."

Semantics of ABoxes:

$\mathcal{I}$ is a model for an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies all assertions in $\mathcal{A}$.

Example: ABox

ABox is a partial description of the world.

ABox $\mathcal{A}$

$\mathsf{Mammal}(\mathsf{dumbo})$

$\mathsf{Darkgrey}(\mathsf{g23})$

$(\mathsf{dumbo}, \mathsf{g23}) : \mathsf{has\text{-}color}$

$\forall \mathsf{has\text{-}color}.\mathsf{Lightgrey}(\mathsf{dumbo})$

$\mathsf{Trunk}(\mathsf{t14})$

$(\mathsf{dumbo}, \mathsf{t14}) : \mathsf{has\text{-}bodypart}$

$(\mathsf{dumbo}, \mathsf{dumbo}) : \mathsf{likes\text{-}most}$

Assertional Reasoning Services

Reasoning tasks for ABoxes:

1. **ABox consistency**
   Given an ABox $\mathcal{A}$ and a TBox $\mathcal{T}$, do they have a common model?

2. **Instance checking**
   Given an ABox $\mathcal{A}$, a TBox $\mathcal{T}$, an individual name a, and a concept C does $a^{\mathcal{I}} \in C^{\mathcal{I}}$ hold in all models of $\mathcal{A}$ and $\mathcal{T}$?

3. **ABox realization**
   Given an ABox $\mathcal{A}$, and a TBox $\mathcal{T}$,
   compute for each individual:
   of which named concepts $C \in \mathcal{T}$ is it an instance of?

Example for ABox Reasoning

ABox is a partial description of the world.

ABox

$\mathsf{Mammal}(\mathsf{dumbo})$

~~$\mathsf{Darkgrey}(\mathsf{g23})$~~

~~$(\mathsf{dumbo}, \mathsf{g23}) : \mathsf{has\text{-}color}$~~

$\forall \mathsf{has\text{-}color}.\mathsf{Lightgrey}(\mathsf{dumbo})$

$\mathsf{Trunk}(\mathsf{t14})$

$(\mathsf{dumbo}, \mathsf{t14}) : \mathsf{has\text{-}bodypart}$

$(\mathsf{dumbo}, \mathsf{dumbo}) : \mathsf{likes\text{-}most}$

TBox

$$\mathsf{Elephant} \equiv \mathsf{Mammal} \sqcap \exists \mathsf{has\text{-}bodypart}.\mathsf{Trunk} \sqcap \forall \mathsf{has\text{-}color}.\mathsf{Grey}$$

$$\mathsf{Grey} \equiv \mathsf{Lightgrey} \sqcup \mathsf{Darkgrey}$$

$$\bot \equiv \mathsf{Lightgrey} \sqcap \mathsf{Darkgrey}$$

1. ABox is inconsistent w.r.t. TBox.
2. dumbo is an instance of Elephant

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Why automated reasoning?

TBox and the ABox capture implicit information.
We want to access this information by making it explicit!

Does my knowledge base . . .

- contain a concept that cannot have instances? (since its definition is contradictory.)
  Check for satisfiabiliy w.r.t. TBox.
- contain an unwanted synonym for a concept? (unwanted / unintended redundancy in my TBox)
  Check for equivalent concepts.
- yield the concept hierarchy I wanted?
  Classify.
- contain individuals not compliant with the specification of the concepts they belong to?
  Check ABox consistency.

Requirements for good reasoning algorithms:

They should be decision procedures,i.e.:

- terminating, — You get always an answer.
- correct, — Every answer is correct.
- complete. — You get all correct answers.

➽ Prerequisit for safe and reliable applications!

Reasoning in $\mathcal{ALC}$

Goal:

Find algorithm for satisfiability and subsumption
(for now: only concept or unfoldable TBoxes
no general TBoxes)

Subsumption can be reduced to (un)satisfiability

$\Longrightarrow$ we can restrict ourselves to satisfiability

Tableau Algorithms

- Used to prove decidability/complexity bounds of DLs
- Most state-of-the-art DL reasoners are based on tableau algorithms

Use the reduction

Reformulate a ...	as an ABox consistency check
satisfiability test: sat(C)?	Consistent: $(\mathcal{T}, \{C(a)\})$?
subsumption test: $C \sqsubseteq_{\mathcal{T}} D$?	Inconsistent: $(\mathcal{T}, \{C \sqcap \neg D(a)\})$?
instance check: $\mathcal{A}, \mathcal{T} \models C(a)$?	Inconsistent: $(\mathcal{T}, \mathcal{A} \cup \{\neg C(a)\})$?

Reasoning method for unfoldable $\mathcal{ALC}$-KBs

1. Use the reduction to reformulate the reasoning problem

2. Expand concepts w.r.t. TBox

3. Normalize concept descriptions

4. Apply tableau rules

Expansion of concept descriptions

$\Longrightarrow$ Reduce satisfiability w.r.t. TBoxes to satisfiability without TBoxes

I.e. get rid of the TBox.

Naive approach for expansion:

Let C_0 be concept, $\mathcal{T}$ unfoldable TBox

1. replace every concept name of a defined concept with the right-hand side of its definitions $A \equiv C$
2. repeat until no more replacements can be made.

Expansion of concept descriptions II

Expansion process terminates due to acyclicity of the concept definitions!

But: exponential blow-up in the worst case!

$$A_0 \equiv \forall R.A_1 \sqcap \forall S.A_1$$
$$A_1 \equiv \forall R.A_2 \sqcap \forall S.A_2$$
$$\vdots$$
$$A_{k-1} \equiv \forall R.A_k \sqcap \forall S.A_k$$

Negation Normal Form

A concept C is in negation normal form (NNF) if negation occurs only in front of concept names.

Transformation rules:

$$\begin{aligned}
\neg\neg C &\rightsquigarrow C \\
\neg(C \sqcap D) &\rightsquigarrow \neg C \sqcup \neg D \\
\neg(C \sqcup D) &\rightsquigarrow \neg C \sqcap \neg D \\
\neg(\exists R.C) &\rightsquigarrow \forall R.\neg C \\
\neg(\forall R.C) &\rightsquigarrow \exists R.\neg C
\end{aligned}$$

Tableau Algorithm: Idea

Try to construct a model for the input concept C_0 as follows:

- Represent models by proof ABoxes
- To decide satisfiability of C_0,
  start with initial proof ABox $\mathcal{A}_{C_0}$
- Repeatedly apply tableau rules
  and check for obvious contradictions
- Return "satisfiable" iff a complete and contradiction-free
  proof ABox was found

Proof ABox

Tableau algorithm works on sets of ABoxes: $\mathcal{S}$

Initially, $\mathcal{S}$ contains proof ABox for concept C_0:

$\mathcal{S} := \{\mathcal{A}_0\}$, with $\mathcal{A}_0 := \{C_0(x_0)\}$

Apply tableau rules to set of proof ABoxes $\mathcal{S}$ until

- a proof ABox is complete or
- there exists an individual x in $\mathcal{A}$ such that $\{B(x), \neg B(x)\} \subseteq \mathcal{A}$ for some concept name B or $\bot(x) \in \mathcal{A}$. (Clash)

Tableau rules for $\mathcal{ALC}$

	Precondition	Replace $\mathcal{A}$ by:
$\longrightarrow_{\sqcap}$	$(C_1 \sqcap C_2)(x) \in \mathcal{A}$ $C_1(x) \notin \mathcal{A}$ or $C_2(x) \notin \mathcal{A}$	$\mathcal{A}' := \mathcal{A} \cup \{C_1(x), C_2(x)\}$
$\longrightarrow_{\sqcup}$	$(C_1 \sqcup C_2)(x) \in \mathcal{A}$ $C_1(x) \notin \mathcal{A}$ and $C_2(x) \notin \mathcal{A}$	$\mathcal{A}' := \mathcal{A} \cup \{(C_1)(x)\}$ $\mathcal{A}'' := \mathcal{A} \cup \{(C_2)(x)\}$
$\longrightarrow_{\exists}$	$(\exists r.C)(x) \in \mathcal{A}$, but no z in $\mathcal{A}$ s.t. $\{r(x,z), C(z)\} \subseteq \mathcal{A}$	$\mathcal{A}' := \mathcal{A} \cup \{r(x,z), C(z)\}$
$\longrightarrow_{\forall}$	$\{(\forall r.C)(x), r(x,y)\} \subseteq \mathcal{A}$, but $C(y) \notin \mathcal{A}$	$\mathcal{A}' := \mathcal{A} \cup \{C(y)\}$

Correctness of the Algorithm

1. The algorithm terminates on any input

2. If the algorithm returns “satisfiable”,
   then the input concept has a model.

3. If the algorithm returns “not satisfiable”,
   then the input concept has no model.

Tableau algorithm for general TBoxes

Treatment of GCIs

1. Code all GCIs into one.

   For $\mathcal{T} = \{ C_1 \sqsubseteq D_1,\ C_2 \sqsubseteq D_2, \ldots,\ C_n \sqsubseteq D_n \}$
   build the GCI $\top \sqsubseteq C_{GCI}$ with
   $C_{GCI} \equiv (\neg C_1 \sqcup D_1) \sqcap (\neg C_2 \sqcup D_2) \sqcap \cdots \sqcap (\neg C_n \sqcup D_n)$

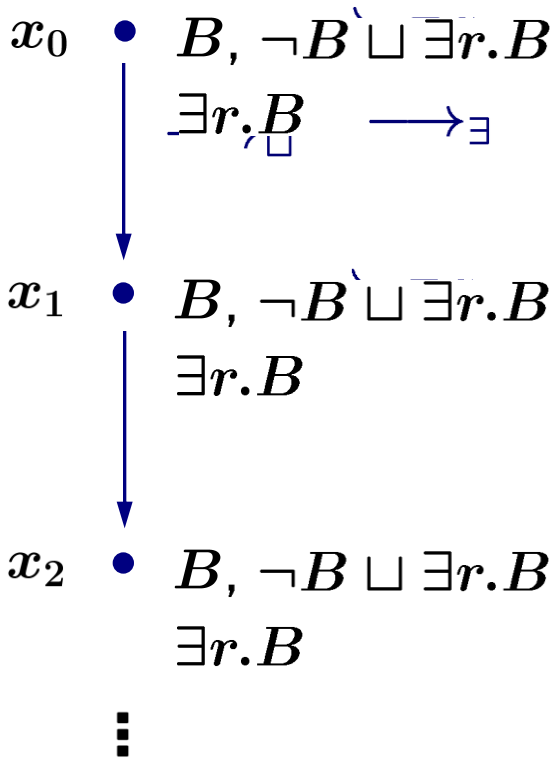
2. New tableau rule: assert C_{GCI} for every individual

   $\longrightarrow_{\top \sqsubseteq C_{GCI}}$: If x in $\mathcal{A}$ and $C_{GCI}(x) \notin \mathcal{A}$,
   then replace $\mathcal{A}$ with $\mathcal{A}' = \mathcal{A} \cup C_{GCI}(x)$

Problem: termination

Consider: $\mathcal{T} = \{B \sqsubseteq \exists r.B\}$

with $C_{GCI} = \neg B \sqcup \exists r.B$



Remedy: Blocking of application of $\longrightarrow_\exists$

An individual x is blocked
by an individual y, iff:

- x was generated by $\longrightarrow_\exists$ after y
- $\{C \mid C(x) \in \mathcal{A}\} \subseteq \{C \mid C(y) \in \mathcal{A}\}$

Tableaux for DLs

For $\mathcal{ALC}$:

- with blocking: termination regained
- Complexity of reasoning w.r.t. general TBoxes: ExpTime-complete

Recall:

- one reasoning method for many reasoning services
- approach requires presence of negation in the DL

Tableau method implemented in most state-of-the-art DL reasoners

For extensions of $\mathcal{ALC}$:

- tableau rules for additional concept constructors

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Expressive power vs. computational complexity

In many cases, the expressive power of $\mathcal{ALC}$ does not suffice:

- an elephant has precisely four bodyparts that are legs
- every elephant has a bodypart which is a trunk
  and every trunk is a bodypart of an elephant

Many extensions of $\mathcal{ALC}$ have been developed, for example:

- qualified number restrictions $(\leq n\ r\ C)$ and $(\geq n\ r\ C)$
- inverse roles r^- to be used in existential and universal restriction

But: Increasing expressivity also increases computational complexity

➽ Tradeoff between expressivity and computational complexity!

Beyond $\mathcal{ALC}$: concept constructors

Number restrictions $(\leq n\, r),\ (\geq n\, r)$

$$(\leq n\, r)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \#\{y \mid (x,y) \in r^{\mathcal{I}}\} \leq n\}$$

$$(\geq n\, r)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \#\{y \mid (x,y) \in r^{\mathcal{I}}\} \geq n\}$$

Qualified number restrictions $(\leq\ n\ r\ C),\ (\leq\ n\ r\ C)$

$$(\leq n\ r\ C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \#\{y \mid (x,y) \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \leq n\}$$

$$(\geq\ n\ r\ C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \#\{y \mid (x,y) \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \geq n\}$$

Example:

$$\text{Car} \sqcap (\geq 5\ \text{has-seat}) \sqcap (\leq 5\,\text{has-seat})$$
$$\sqcap\ (\geq 1\ \text{has-seat Drivers-seat}) \sqcap (\leq 1\ \text{has-seat Drivers-seat})$$

Beyond $\mathcal{ALC}$: Concept constructors II

Sometimes it is useful to refer to individuals in the TBox.

Recall: If they have same description

- concepts are equivalent.
- individuals are distinct.

C $\equiv$ ($\forall$ has-child.$\bot$)
D $\equiv$ ($\leq$ 0 has-child)
$\Longrightarrow$ C $\equiv$ D

(Carla, Luisa): parent, Person(Carla),
(Markus, Luisa): parent, Person(Markus)
$\Longrightarrow$ Carla $\not\equiv$ Markus

Concept constructors using individuals:

- Nominals $\{a\}$ $\quad\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$
- One-of $\{a_1, \ldots, a_n\}$ $\quad\{a_1, \ldots, a_n\}^{\mathcal{I}} = \{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$

E.g.: RomanCatholic $\sqsubseteq$ $\exists$ knows.{Pope}

Beyond $\mathcal{ALC}$: Roles

Role declarations

R	atomic role	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ e.g. has-child
F	feature or attribute	$F^{\mathcal{I}} = \{(x,y) \mid (x,y) \in F^{\mathcal{I}} \wedge (x,z) \in F^{\mathcal{I}} \Rightarrow y = z\}$ e.g. has-mother
$R \sqsubseteq S$	role inclusion role hierarchy	$R \sqsubseteq S$ holds in $\mathcal{I} \Leftrightarrow R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$

e.g. has-mother $\sqsubseteq$ has-parent $\sqsubseteq$ has-family-member

has-sibling $\sqsubseteq$ has-family-member

Beyond $\mathcal{ALC}$: Roles II

Role declarations

R^+ transitive Role $(R^+)^{\mathcal{I}} = \{(x,z) \mid (x,y) \in R^{\mathcal{I}}, (y,z) \in R^{\mathcal{I}} \Rightarrow (x,z) \in R^{\mathcal{I}}\}$

e.g. has-ancestor

R^- inverse Role $(R^-)^{\mathcal{I}} = \{(y,x) \mid (x,y) \in R^{\mathcal{I}}\}$

e.g. $(\text{has-parent})^- = \text{has-child}$

$S \circ R \sqsubseteq R$ right identities $(x,y) \in S^{\mathcal{I}} \wedge (y,z) \in R^{\mathcal{I}} \rightarrow (x,z) \in R^{\mathcal{I}}$

e.g. has-part $\circ$ has-function $\sqsubseteq$ has-function

Excursus: names of description logics

Basis-DL: $\mathcal{ALC}$

- $\mathcal{E}$: Existential restrictions
- $\mathcal{N}$: Number restrictions
- $\mathcal{Q}$: Qualified number restrictions
- $\mathcal{O}$: nominals, Objects
- $\mathcal{F}$: Features, functional roles
- $^+$: Transitive roles
- $\mathcal{I}$: Inverse roles
- $\mathcal{H}$: role Hierarchies
- $\mathcal{R}$: right identities

$\mathcal{S}$: Abbreviation for $\mathcal{ALC}^+$

Complexity of subsumption in some DLs

	unfoldable TBox	general TBox
$\mathcal{EL}$ $(\sqcap, \exists)$	P	P
$\mathcal{ALC}$	PSPACE	EXPTIME
$\mathcal{ALCNIO}$	EXPTIME	NEXPTIME
$\mathcal{SHOIQ}$	NEXPTIME	NEXPTIME
$\mathcal{SROIQ}$		2-NEXPTIME

- These are worst case complexities!
  - worst case does not need to appear in practice
  - worst case requires regular structure of the KB

Tableaux reasoner systems for DLs

- Fact++ — $\mathcal{SROIQ}$
- Pellet — $\mathcal{SHOIQ}$
- RacerPro — $\mathcal{SHIQ}$
- Hermit — $\mathcal{SHIQ}$
  employs Hypertableaux
- CB — Horn-$\mathcal{SHIF}$
  consequence-based reasoner — fragment of OWL 2

Most systems offer more reasoning functionality

Complexity of subsumption in some DLs

	unfoldable TBox	general TBox
$\mathcal{EL}$ $(\sqcap, \exists)$	P	P
$\mathcal{ALC}$	PSPACE	EXPTIME
$\mathcal{ALCNIO}$	EXPTIME	NEXPTIME
$\mathcal{SHOIQ}$	NEXPTIME	NEXPTIME
$\mathcal{SROIQ}$		2-NEXPTIME

- OWL DL
- OWL 2

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Light-weight DL: $\mathcal{EL}$

$\mathcal{EL}$ $(\sqcap, \exists, \top)$ is used in medical and biological ontologies

$$\mathsf{Inflammation} \sqsubseteq \mathsf{Disease} \sqcap \exists \mathsf{acts\text{-}on}.\mathsf{Tissue}$$

$$\mathsf{HeartDisease} \equiv \mathsf{Disease} \sqcap \exists \mathsf{has\text{-}loc}.\exists \mathsf{comp\text{-}of}.\mathsf{Heart}$$

$$\mathsf{Retina} \sqsubseteq \mathsf{Tissue} \sqcap \exists \mathsf{has\text{-}loc}.\mathsf{Eye}$$

The $\mathcal{EL}$ family

Prominent members:

$\mathcal{EL}$: $\quad \sqcap, \exists, \top$

$\mathcal{EL}^+$ extends $\mathcal{EL}$ by:
- role chain inclusions: $r \circ s \sqsubseteq t$.
  e.g. has-mother $\circ$ has-sister $\sqsubseteq$ has-aunt.)

$\mathcal{EL}^{++}$ extends $\mathcal{EL}^+$ by:
- $\bot$
- nominals
- corresponds to OWL 2 EL profile

Typically, used with general TBoxes!

Subsumption in $\mathcal{EL}$

Steps of the subsumption algorithm for $\mathcal{EL}$:

1. Normalize TBox
2. Translate normalized TBox into "Completion Sets"
3. Saturate sets using completion rules
4. Read off subsumption relationships from completeion sets

TBox Normalization I

Assumptions:

- consider only the subsumption of concept names w.r.t. TBoxes

  $C \sqsubseteq_{\mathcal{T}} D$ iff $A \sqsubseteq_{\mathcal{T}'} B$, where $\mathcal{T}' = \mathcal{T} \cup \{A \equiv C, B \equiv D\}$

- TBox statements always have the form $C \sqsubseteq D$

  $C \equiv D$ equivalent to $C \sqsubseteq D, D \sqsubseteq C$

Normalized TBox only has statements of the following form:

$$A \sqsubseteq B$$
$$A \sqcap A' \sqsubseteq B$$
$$A \sqsubseteq \exists R.B$$
$$\exists R.A \sqsubseteq B$$

A, A', B: concept names or $\top$

TBox Normalization II

Normalization can be achieved by introducing new concept names:

Let $C \sqsubseteq D \in \mathcal{T}$ with C, D complex concepts

Top-most operator in D (C treated analogously):

- $D = \exists R.E$

  $C \sqsubseteq D \quad \rightsquigarrow \quad A \sqsubseteq E,\ E \sqsubseteq A,\ C \sqsubseteq \exists R.A$

- $D = E \sqcap F$

  introduce "new names" for E and F

Additional case:

$C \sqsubseteq D \sqcap E$ replaced by $C \sqsubseteq D$ and $C \sqsubseteq E$

Completion Sets

Compute a completion set $S(A)$ for every concept name A in $\mathcal{T}$ (and every existential restriction)

Intuition:

- $S(A)$ contains all concept names B with $A \sqsubseteq_{\mathcal{T}} B$
- $S(A, r)$ contains all concept names B with $A \sqsubseteq \exists r.B$

Initially:

- $S(A) = \{A, \top\}$ for each concept name A
- $S(A, r) = \emptyset$ for each concept name A and role name r

Completion rules

Then saturate completion sets using the rules:

CR1 $A' \sqsubseteq B \in \mathcal{T}$, $A' \in S(A)$, $B \notin S(A)$ $\rightsquigarrow$ add B to $S(A)$

CR2 $A_1 \sqcap A_2 \sqsubseteq B \in \mathcal{T}$, $A_1, A_2 \in S(A)$, $B \notin S(A)$ $\rightsquigarrow$ add B to $S(A)$

CR3 $A' \sqsubseteq \exists r.B \in \mathcal{T}$, $A' \in S(A)$, $B \notin S(A, r)$ $\rightsquigarrow$ add B to $S(A, r)$

CR4 $\exists r.C' \sqsubseteq B \in \mathcal{T}$, $C' \in S(C)$, $C \in S(A, r)$, $B \notin S(A)$ $\rightsquigarrow$ add B to $S(A)$

Completion Sets II

Lemma. 1. $B \in S(A)$ iff $A \sqsubseteq_{\mathcal{T}} B$.

2. subsumption sets can be computed in polynomial time.

ad 2.

- there are at most $|\mathcal{T}|$ completion sets to be computed;
- each completion set contains at most $|\mathcal{T}|$ elements

$\Longrightarrow$ at most $|\mathcal{T}|^2$ rule applications

- searching for an applicable rule needs time at most $|\mathcal{T}|^2$

$\Longrightarrow$ algorithm runs in time $|\mathcal{T}|^4$

Completion Sets for ABox Realization

Compute a completion set $S(A)$ for every

- concept name A in $\mathcal{T}$ (and every existential restriction),
- every individual (and every role successor)

Intuition:

- $S(A)$ contains all concept names B with $A \sqsubseteq_{\mathcal{T}} B$
- $S(A, r)$ contains all concept names B with $A \sqsubseteq \exists r.B$
- $S(a)$ contains all concept names B with a instance of B
- $S(a, r)$ contains all concept names B with a instance of $\exists r.B$ (and individuals b with $r(a, b) \in \mathcal{A}$)

Completion-based reasoners for $\mathcal{EL}$

CEL (Classifier for $\mathcal{EL}$)

- supported DL: $\mathcal{EL}+$
- for "good natured" TBoxes: incremental classification
- explanation of subsumption relations (axiom pinpointing)

jCEL (Java classifier for $\mathcal{EL}$)

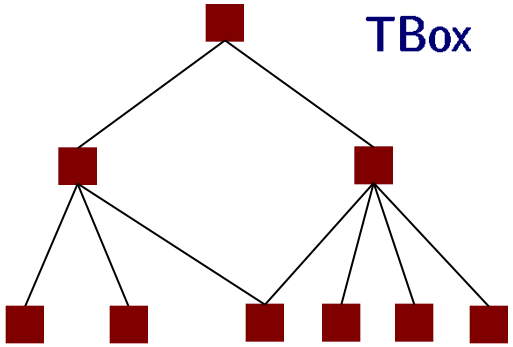
- supported DL: $\mathcal{ELHI}f_{\mathcal{R}}$
- TBox classification + ABox realization

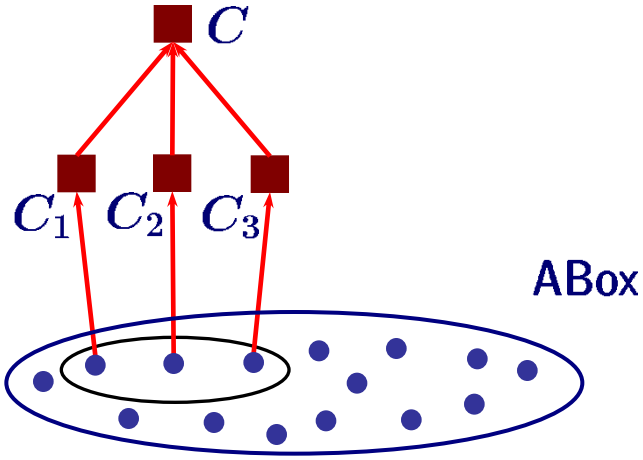
Bottom-up construction of KBs

Idea:

Allow the knowledge engineer to define concepts by giving examples:

1. examples given by ABox individuals
2. for each example, compute the most specific concept describing the individual
3. extract their commonalities by computing the least common subsumer





Generalizations in $\mathcal{EL}$

LCS does not always exist in the presence of general TBoxes:

$$A \equiv \exists R.A$$
$$B \equiv \exists R.B$$

LCS(A, B) w.r.t. $\mathcal{T}$: ???

Approach without TBoxes:

1. Represent $\mathcal{EL}$-concepts by $\mathcal{EL}$-trees
2. Compute the LCS as the product of $\mathcal{EL}$-trees.

Approximative solution for general $\mathcal{EL}$-TBoxes:

1. Compute the LCS up to a fixed role-depth.
2. Use the completion sets to generate the $\mathcal{EL}$-trees

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Light-weight DL: DL-Lite family

DL-Lite$_{core}$

- concept constructors: $A, \exists r.\top, \exists r^-.\top$
- roles: inverse
- TBox axioms: $C \sqsubseteq D$

DL-Lite$_{\mathcal{F}}$ extends DL-Lite$_{core}$ by:

- (inverse) functional roles

DL-Lite$_{\mathcal{R}}$ extends DL-Lite$_{core}$ by:

- role hierarchy
- role disjointness axioms

. . . for roles and their inverse

Core of the OWL 2 QL profile

DL-Lite family

- designed for ontology-based data access
- tailored towards applications that need to handle huge amounts of data
- allow efficient querying of ABoxes (w.r.t. a fairly light-weight TBox)

➳ required to store ABox in relational data base system and use relational DB engine for querying

Conjunctive Queries

Definition:

Conjunctive Query $q(X)$ w.r.t. a KB $= (\mathcal{T}, \mathcal{A})$ has the form:

$$q(X) \longleftarrow \exists Y.conj(X, Y).$$

where

X : is a tuple of answer set variables

Y : is a tuple of free variables

$conj$: is a set of expressions of the form:
$C(z_i)$ or $r(z_i, z_j)$, where C:
concept description, r: role and $\{z_i, z_j\} \subseteq X \cup Y$.

➳ Limited form of FOL queries

Conjunctive Queries

Query answering:

the answers to a query q w.r.t. a KB $\mathcal{K}$ is
the set of tuples of individuals from $\mathcal{K}$ $(a_1, \ldots, a_n)$ such that

$$(a_1^{\mathcal{I}}, \ldots, a_n) \in q\mathcal{I}$$

for all models of $\mathcal{K}$.

"Compute tuples from $\mathcal{K}$ for which the query q always evaluates to true."

Recall:

- "traditional" ABox queries are concept-based
  They allow only for tree-shaped queries.

  Example.: Query for all Persons, who are their best friend.

  ➳ use of variables!

- Conjunctive queries are a more flexible query language!

Approach for query answering in DL-Lite

1.) Use TBox to reformulate query q into a FOL-query $q_{\mathcal{T}}$, discard the TBox

"Compile TBox information into the query."

2.) View ABox $\mathcal{A}$ as a relational database $\mathcal{I}_{\mathcal{A}}$, where
- $A^{\mathcal{I}_{\mathcal{A}}} = \{a \mid A(a) \in \mathcal{A}\}$
- $r^{\mathcal{I}_{\mathcal{A}}} = \{(a, b) \mid r(a, b) \in \mathcal{A}\}$

3.) Evaluate $q_{\mathcal{T}}$ in the DB $\mathcal{I}_{\mathcal{A}}$ using a relational query engine.

Excursus: complexity

Complexity classes: definied w.r.t. size of the input.
Ontology-based applications:
Sometimes the TBox is small compared to the ABox!

Query complexity:

complexity w.r.t. number of conjunct of the query.

Data complexity:

complexity w.r.t. size of the ABox.

Combined complexity:

:= complexity w.r.t. size of the TBox and the ABox.

Complexity for query answering for conjunctive queries

Expressive DLs:

- $\mathcal{SHIQ}$
  combined complexity: 2ExpTime-complete
- $\mathcal{ALCI}$
  combined complexity: 2ExpTime-complete

light-weight DLs:

- $\mathcal{EL}$:
  NP-hard für combined complexity
- $\mathcal{DL}$-Lite:
  PTime in size of TBox
  LogSpace for data complexity

DL Lite family in practice

- allows for efficient ontology-based data access
- DL Lite$_{\mathcal{R}}$: maximal DL with LogSpace data complexity
- query answering for DL Lite$_{\mathcal{R}}$ is implemented in QuOnto
- DL Lite$_{\mathcal{R}}$: core of OWL 2 QL

Conclusions

- DLs offer a variety of ontology languages to model application domains
- DLs have formal semantics which is used to define reasoning services
- DL systems offer powerful reasoning services
- OWL 2 offers 3 DL-based ontology languages
- OWL 2 offers 2 lightweight ontology languages for which reasoning scales well
- Semantic Web applications are needed to demand / inspire new ontology services